WAVELET DE-NOISING FOR AUTONOMOUS LATITUDE DETERMINATION

Autonomous determination of the latitude of the place of movable and immovable objects is used as an independent task, as well as the task of determination of the initial value of latitude for operation of both platform and platform-free navigation systems. To solve these problems, it is necessary to have an inertial measurement unit (IMU) with at least three gyroscopes and three accelerometers. When using the IMU, executed by MEMS technology, the output signals of micromechanical gyroscopes and accelerometers have significant noise components. Kalman filter is usually used to filter such signals. However, for this purpose it is necessary to know, besides the exact mathematical model of sensitive elements, many of their initial random characteristics.

In the article, the research was conducted in order to investigate the use of wavelet transformation for the filtering of output signals of micromechanical accelerometers and gyroscopes for autonomous determination of the latitude of the place. The peculiarity of using wavelet transform for noisy signals is that due to changing scale, wavelets can detect differences in process characteristics on different scales, and with help of the shift we can analyze process properties at different points on the whole investigated interval. Due to the properties of this system's fullness that it is possible to restore the process by means of inverse wavelet transform. The efficiency of the developed method of increasing the accuracy of the autonomous determination of the latitude of the IMU on the basis of micromechanical gyroscope and accelerometers has been experimentally confirmed. The projections of the angular velocity of Earth rotation and gravitational acceleration were obtained from the IMU made by MEMS technology. After that, the signals of the gyroscopes and accelerometers of the inertial measuring unit were filtered, using the wavelet ‘Daubechies 10’ in decomposition, and averaged. These signals were used in a computational algorithm to determine the latitude. The results showed that, unlike the well-known Kalman filter, which almost did not increase the accuracy of the latitude calculation, wavelet denoising and further averaging reduced calculation error by almost twice.

Keywords: gyroscopes; accelerometers; latitude determination; wavelets.
The study is carried out by simulating the operation of a gyromagnetic compass. The root mean square error (rms) and the mean value of the error are taken as the correction characteristics. At the same time, the transitional process of the initial exhibition is also controlled.

In the algorithms under study, new solutions are applied.

In the differential PID channel, a quarter-period delay of the dominant oscillations is applied, and the gain of the differential channel is adjusted according to the oscillation frequency. This setting allows you to almost completely smooth out the oscillatory error of the gyromagnetic compass by compensating for the oscillatory error with the received signal in the differential circuit.

In the Kalman filter scheme, the resulting heading error estimate is in antiphase with the error. After the introduction of a delay in the estimate for half a period of fluctuations, the estimate almost completely corresponds to the error. As a result, the oscillatory error can be almost completely eliminated in the instrumental heading value.

The article shows that all three investigated correction schemes show better characteristics in comparison with the known basic scheme. The highest accuracy can be achieved when applying the Kalman filter with the necessary settings for the perturbation, observation and initial error matrices.

The simplest to implement is a circuit with an adaptable PID controller. Its characteristics are close to the scheme with the Kalman filter.

**Keywords:** gyromagnetic compass; correction circuit; proportional-integral-differential controller; fuzzy controller; Kalman filter.

### Introduction

The magnetic channel was and remains in demand for the correction of gyroscopic systems: both in gyromagnetic compasses (GMC) proper and in corrected inertial navigation systems and in other developments. This correction was carried out both many years ago and now. The relevance of such a channel has grown recently in connection with the progress in the development of micromechanical gyroscopes and accelerometers, micromagnetic sensitive elements.

Basically, the magnetic channel is used to correct the heading instrument readings, since other reference systems (satellite navigation system, Doppler meters, etc.) still rarely provide information about the heading of the object. In addition, satellite navigation systems have insufficient noise immunity. The satellite signal can be lost in the shade of buildings, trees, or under the influence of artificial interference.

Magnetic correction attracts with its simplicity and autonomy. The active use of the magnetic channel is also due to the increased capabilities of signal processing with modern computing means.

A gyromagnetic compass is a control system in which all modern methods and schemes are applicable. Its main characteristics are the accuracy and time of the initial alignment (transition process).

GMC errors are divided into static (systematic) and dynamic. Static errors (e.g. magnetic deviation) are eliminated by calibration and compensation. Dynamic errors are eliminated by using various regulators.

The study of the modern methods and algorithms possibilities for using to improve the characteristics of the gyromagnetic compass is carried out in the presented article.

### Problem statement

Many existing marine and aviation movable objects are equipped with heading systems, in which the heading channel is corrected by a magnetic heading sensor. Such systems are called gyromagnetic compasses (for example, gyromagnetic compass GMK-1, attitude and heading reference systems (AHRS) LCR-100). Magnetic heading correction is also widely used in modern AHRS with micromechanical gyroscopes [1].

A known means of achieving good dynamic performance is the PID controller. In the literature there are enough examples of its application to thermal processes, various types of drives, machine tool control, autopilots [2, 3, 4]. Proportional-integral (PI) controllers of the GMC are considered in detail in [5], in inertial navigation systems (INS) [6]. In this case, the use of a differential channel in orientation devices is not considered. An exception is the example of using a PID controller at the initial alignment of INS [7]. The article [8] and [9] shows the features of the use of the PID con-troller in the gyromagnetic compass.

A well-known means of improving the performance of a control system is the use of fuzzy controllers (fuzzy con-trollers). In [10, 11], the advantages of tuning the observation matrix of the Kalman filter using a fuzzy controller are shown in terms of the parameters of the disturbances experienced by gyroscopes, accelerometers, and magnetometers. In [12], the possibility of adaptive tuning of the generalized Kalman filter in conditions of instability of the calculated scale factors of the model of a strapdown inertial navigation system (SINS) is shown.

In [13], the application of a Fuzzy controller is shown to correct the readings of the integrated navigation inertial-satellite system according to the parameters of the maneuver performed by the object. In this case, 3 triangular mem-bership functions are used at the input and output of the regulator, which
operates according to 9 rules. PhD thesis [14] is devoted to the study of the characteristics of a cheap (inexpensive) heading standard (AHRS) using Fuzzy controllers and Kalman filters.

In [15], the accuracy of ship control by a control system with a PID controller and a fuzzy controller of varying complexity (9, 11 or 13 membership functions) is investigated. It is shown that fuzzy controllers give a higher navigation accuracy compared to navigation with a PID controller.

Modern methods for determining and eliminating magnetic deviations are considered in [16]. The application of the variant of the generalized Kalman filter - UKF, as well as the Kalman filter with strong tracking (STUKF) is investigated in detail.

The magnetic correction channel in the GMC uses a regulator with proportional or proportional-integral correction [5]. Recently, other controllers have been actively used: PID controllers, fuzzy controllers, controllers with a Kalman filter. Correction schemes with such regulators are the object of research.

There is a need to study what advantages, primarily in terms of dynamic accuracy, can be obtained from their application.

The purpose and objectives of the study
The research tasks are modernization, determination of the capabilities, characteristics, comparative analysis of various correction schemes for the gyromagnetic compass (magnetic correction channel) in order to develop recommendations for their use.

Materials and research methods
The research will be carried out by modeling correction schemes in the Matlab package. As a basis for comparison, let us take a well-known scheme with integral-positional (integral-proportional) correction [5]. This scheme has variations. Correction can be moment or kinematic. The correction can have a tracking loop or it can be built according to the compensation scheme. The latter is sometimes called a complementary filter.

Integral-positional correction scheme
Let us take as a basis a circuit with a moment integral-positional (isodromic, proportional-integral, PI) correction with a tracking loop. Its modeling scheme in the Simulink package follows from Fig. 1, if the differential channel is excluded in the lower part.

In Fig. 2:
- km – magnetic heading,
- kmp – instrumental heading value (gyromagnetic heading),
- dkm – instrument heading error,
- SineWave – block for setting harmonic interference,
- \( k_{dp} \) – transmission coefficient of the angle sensor,
- \( ku \) – the gain of the proportional correction loop,
- \( ki \) – transmission coefficient of the integral correction contour,
- \( 1/s \) – integrator,
- \( kdm \) – coefficient of transmission of the torque sensor,
- \( H \) – the kinetic moment of the gyroscope,
- \( Mp \) – gyroscope outrageous moment (interference),
- al0 – initial misalignment of magnetic and gyroscopic headings,
- \( omdz \) – the vertical component of the angular velocity of the accompanying trihedron.

The dash-dotted line marks the knot of the regulator, in which changes are assumed.

The graphs of errors and control signals are shown in Fig. 2. In Fig. 2
- \( dkp, dmf \) – gyromagnetic heading error,
- \( kmf, kmf \) – gyromagnetic heading,
- \( integr \) – voltage at the output of the regulator integrator,
- \( Uupr \) – control voltage at the torque sensor input,
- \( Afm, omf \) – interference amplitude and frequency of the magnetic heading sensor,
- \( f = A_{fm} \sin \omega t \),
- \( h \) – integration step,
- \( rmsp \) and \( averp \) are the root mean square error and the mean value of the gyromagnetic heading error for the circuit with a PI controller for the last 60 s.

As can be seen from Fig. 2, the graphs show the presence of a transient process for 60 s, due, first of all, to the work of the integrator.

As shown in [4], the output signal of the SMC can be described by the dependence

\[
\begin{align*}
    k_{uv} &= k_u + \frac{T_u p + 1}{TT_u p^2 + T_u p + 1} f + \frac{T_u p}{k_u (TT_u p^2 + T_u p + 1)} \Delta v + \frac{TT_u p^2}{TT_u p^2 + 1} \left[ \alpha_y + \alpha(0) \right],
\end{align*}
\]

where
- \( T = H / k_{ab} k_{sy} k_{dm} \) – the time constant,
- \( T_i = 1/k_i \) – the time constant of the integrator,
- \( \alpha_y \) – the dynamic and kinematic drift of the gyroscopic and magnetic headings,
- \( \Delta \) – the zero offset of the integrator.

GMC errors depend on the interference \( f \) of the magnetic heading sensor, the moment-interference of the gyroscope \( Mp \), the error of the initial alignment and the kinematic errors of the gyroscopic (cardan errors, errors in the conical motion of the object, etc.), and the integrator error. Other instrumental errors also contribute.
Adaptive PID controller circuit

Attention is drawn to the use of a proportional-integral-derivative (PID) controller. In [8], the features of using the differential channel in the PID controller of the GMK are shown. With the help of a differential channel with a phase shift of 90 degrees, the oscillatory component of the magnetic compass interference segregate and compensate. In Fig. 2, the lower part simulates a GMC circuit with adaptable PID controller. The upper part corresponds to the GMC version with a different type of nonlinear controller.

When using the PID controller, in the differential channel the variable interference is segregate and the systematic component of the signal is removed.

To eliminate variable interference, a quarter-period delay is introduced into the derivative of the interference heading signal \( \tau = \pi / (2 \cdot \omega_0) \), see Fig. 3.

The transfer coefficient of the differential circuit \( kd/omf \) depends on the frequency of the disturbance of the magnetic sensor, which follows from the expression

\[
\frac{d}{dt} A_{f_{m} \sin \omega_0 t} = A_{f_{m} \omega_0} \cos \omega_0 t .
\]
If the coefficients are $kd = ku$, we obtain an alternating signal of the differential circuit, corresponding in magnitude and in antiphase with the oscillatory interference. The addition of the signals of the proportional and differential circuits practically eliminates the influence of variable interference (Fig. 4).

To determine the required delay $\tau$ and adjust the transmission coefficient of the differential circuit, it is necessary to determine in advance the oscillation period (angular frequency) of the interference.

The result of modeling a GMC with a PID controller is shown in Fig. 4 (blue solid lines). As can be seen from the $dkp$ graph and the value of the root mean square error $rmsp = 3.58$ arc.min, The oscillatory noise is almost completely eliminated. Average error value for the last 60 s $averp = -1.56$ arc.min.

In [8], it was shown that even with a random pitching with a dominant frequency, a circuit with a PID controller gives a noticeable positive result.

**Fuzzy controller circuit**

When designing a fuzzy controller, the range of tools is great [12]. If we use the simplest controller, built according to the Mamdani type, with three triangular membership functions (terms) of both the input signal (fuzzification) and the output signal (defuzzification), then by choosing the width of the windows, the location and width of the terms, we can obtain very different types static characteristics of controllers. Fig. 5 shows the Fuzzy Logic Designer package windows from Matlab 2017b.

With the parameters of the input terms (input) indicated in Fig. 6 a and the parameters of the output terms (output) in Fig. 6 c, we obtain the characteristic shown in Fig. 7 a. The errors of the circuit with a Fuzzy controller ($rmsf$, $averf$) shown in Fig. 4 correspond to this setting.

In the Rule Viewer window (Fig. 5) you can control the operation of the regulator, make sure that

[Fig. 3. Signal at the output of the proportional loop (dashed) and differential loop (solid)]

[Fig. 4. PID controller ($dkp$, $kmp$, $intp$, $Uuprp$) and Fuzzy controller ($dkf$, $kmf$, $intf$, $Uuprf$) circuit characteristics]
Thus, the adjustment of a Fuzzy controller is an expert and requires a certain experience and skill of the designer. As can be seen from Fig. 4, a Fuzzy controller allows one to obtain a smaller average error value $\text{averf} = -0.78 \text{ arc.min.}$ in comparison with the average value $\text{averp} = -1.56 \text{ arc.min.}$ when using a PID controller or $\text{averp} = -2.03 \text{ arc.min.}$ when using a PI controller. It is likely that better results can be obtained with a more complex fuzzy controller structure.

Fig. 5. Window **Rule Viewer**

![Input window](image1)

* a - input window  

![Output window 1](image2)

* b - output window 1

![Output window 2](image3)

* c - output window 2

![Controller rules](image4)

* d - controller rules

Fig. 6. Fuzzy controller design windows
Correction scheme with Kalman filter

The Kalman filter (KF) is usually used to correct the heading channel of an inertial navigation system (INS) to a magnetic heading. A closed gyromagnetic channel with a Kalman filter is shown in Fig. 8, where MC is a magnetic compass.

In the absence of correction in the considered short time interval 2.5 min. (much less than the Schuler period) the heading error (Fig. 9) for the INS parameters indicated in Fig. 10 changes from +6 arc.min. (initial alignment error) to -4 ang.min. This change, as is known [17], will continue in the future. Magnetic compass correction will eliminate this build-up of heading error. If a magnetic compass of an analytical type is used with magnetometers rigidly installed on board, integrated with an INS, its error will mainly have a noise component. If a magnetic compass of a semi-analytical type with a pendulum suspension of magnetometers is used for correction, a pronounced oscillatory error is possible, which was considered in the previous schemes.
Let us consider the application of the linear discrete KF, the algorithm of which is well known and presented in [17].

When using the KF in a closed circuit (Fig. 8), the experimental conditions and error characteristics shown in Fig. 9 (there is no oscillatory error of the magnetic heading sensor), the heading angle error corresponds to the dK graph.

In Fig. 9
fi0 - latitude,
dtet0, dgam0 - initial leveling errors,
dK0 - gyrocompass error,
V - speed of movement,
omr - systematic error of gyroscopes,
da - systematic error of accelerometers,
h - the integration step,
k, tet0, gam0 - angles of heading, pitch and roll,
sgx - root-mean-square deviation (sko) of the gyroscope noise (1),
sigm - sko of accelerometer noise,
sigmz - sko of the magnetic heading sensor error,
kRm, kQ, kP, kOm, kAc - FK tuning factors,
bo, ba, bu - signs of inclusion of feedbacks on the systematic drift of gyroscopes, accelerometers, heading angle, respectively.

Note that in this scheme, the KF tuning is applied by multiplying the initial error matrix P by kP = 10, and the measurement noise matrix R by kR = 0.1. This setting results in a reduction in transient time.

kOm and kAc are the weight coefficients of the noise of the gyroscopes and accelerometers, respectively, in the perturbation matrix Q.

It is possible to estimate the error and obtain the gyromagnetic heading with an error within 0.5 arc.min. (Fig. 10). In Fig. 10, 11 the upper window shows the INS heading error and its estimate by the Kalman filter, the lower window shows the error with which the estimate was made (estimation error).

Recall that in this study errors from magnetic deviations, which are classified as systematic errors, are not taken into account.

In the presence of an oscillatory error, we get the errors in Fig. 11. The error estimate is in antiphase with the INS error. The transient process of the error with the performed filter settings takes about 60 s. By introducing a delay of half the oscillation period, we can synchronize the heading error and its estimate. In Fig. 11, the upper window shows the INS heading error with an interference amplitude of the magnetic sensor of 5.7 degrees, its estimation using the magnetic heading sensor after entering the delay. The lower window shows the estimation error in the presence of a delay. The error is within 1 arc.min, but this requires a sufficiently accurate delay with an error of no more than 0.1 s. In practice, given the randomness of fluctuations, such a result is hardly achievable.

**Research results and their discussion**

The simulation results are summarized in Table 1. It shows the error characteristics of the gyromagnetic compass for the well-known proportional-integral correction scheme, adaptable proportional-integral-differential correction, schemes with a fuzzy controller and a Kalman filter.
The initial exposure took about 1 min. are taken as characteristics. The transient error process at periods (1 min). Of the change in the quasiharmonic error of the error (average) in the steady state for the last 5

Table 1. Simulation results for different correction schemes

<table>
<thead>
<tr>
<th></th>
<th>proportional-integral (PI)</th>
<th>Adaptive proportional-integral-differential (PID)</th>
<th>Fuzzy controller</th>
<th>Kalman filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>rms, arc.min</td>
<td>44</td>
<td>3.4</td>
<td>34</td>
<td>0.67</td>
</tr>
<tr>
<td>average, arc.min</td>
<td>-2.1</td>
<td>-1.6</td>
<td>-0.8</td>
<td>0.14</td>
</tr>
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</table>

Conclusions

The simulation of four possible options for correcting the gyromagnetic compass shows that correction schemes with an adaptable PID controller, Fuzzy controller, Kalman filter show better characteristics in comparison with the widely used scheme with proportional-integral (integral-positional, isodromic) correction. New in the PID controller scheme is the input of the time delay and the setting of the differential loop gain. New in the scheme with the Kalman filter is delay introduced for half a period of the oscillatory component. It is useful to make a comparison of these circuits after research with real signals.

The simplest to implement in the GMC correction scheme is an adaptable proportional-integral-derivative (PID) controller.

The studies carried out, the proposed modernization of correction circuits can be used to improve the characteristics of existing or newly developed gyromagnetic compasses.

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СХЕМИ КОРІНКІ ГІРОМАГНІТНОГО КОМПАСУ

У зв'язку зі значним прогресом у вдосконаленні чутливих елементів приладів (гіроскопів, акселерометрів, магнітометрів), а також збільшенням можливостями обробки інформації обчислювальними засобами виникла необхідність у застосуванні сучасних досконалих алгоритмів побудови систем корекції гіромагнітного компасу. Об'єктом дослідження є схеми корекції з адаптивним пропорційно-інтегрально-диференціальним (ПІД) регулятором, з регулятором з нечіткою логікою (Fuzzy controller), з фільтром Калмана для корекції гіроскопічного вимірювача за значеннями магнітного датчику курсу. Водночас, за основу для порівняння береться відома схема гіромагнітного компасу з ментурно пропорційно-інтегральною корекцією. Розглядається згладжування коливальної помилки магнітного компаса, яка може бути переважаючою.

Предмет дослідження - характеристики точності за усталеного режиму. Дослідження проводиться за умови моделювання роботи гіромагнітного компасу. Як характеристики корекції прийнята середня квадратична помилка (rms) і середня значення помилки. Водночас, характеризуется також переходный процес початкової вставки. У досліджуваних алгоритмах застосовано нові рішення. У диференціальному каналу ПІД застосовано затримку на чверть періоду коливань, а коефіцієнт передачі диференціального канала налаштовується за часотою коливань. Подібне настроювання дозволяє майже повністю згладити коливальні помилки гіромагнітного компаса за умови компенсації коливальної помилки отриманим сигналом диференціального контура. У схемі з фільтром Калмана отримано оцінку помилки курсу перебуває у противідніз помилкою. Після введення затримки оцінки на півперіоду коливань оцінка практично повністю відповідає помилки. Таким чином в приладному значенні курсу можна майже повністю усунути коливальну помилку.

У статті показано, що всі три досліджувані схеми корекції показують кращі характеристики порівняно з відомою базовою схемою. Найбільшою точністю можна досягти при застосуванні фільтра Калмана з необхідними налаштуваннями матриць збурення, вимірювання та матриці початкових помилок. Найбільшою особливістю для реалізації є схема з адаптованим ПІД-регулятором. Її характеристики зблизькі до схеми з фільтром Калмана.

Ключові слова: гіромагнітний компас; схема корекції; пропорційно-інтегрально-диференціальний регулятор; нечіткий контролер; фільтр Калмана.